LETTERS TO THE EDITOR.

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Reflection of Light at the Confines of a Diffusing Medium.

I SUPPOSE that everyone is familiar with the beautifully graded illumination of a paraffin candle, extending downwards from the flame to a distance of several inches. The thing is seen at its best when there is but one candle in an otherwise dark room, and when the eye is protected from the direct light of the flame. And it must often be noticed when a candle is broken across, so that the two portions are held together merely by the wick, that the part below the fracture is much darker than it would otherwise be, and the part above brighter, the contrast between the two being very marked. This effect is naturally attributed to reflection, but it does not at first appear that the cause is adequate, seeing that at perpendicular incidence the reflection at the common surface of wax and air is only about 4 per cent.

A little consideration shows that the efficacy of the reflection depends upon the incidence not being limited to the neighbourhood of the perpendicular. In consequence of diffusion[1] the propagation of light within the wax is not specially along the length of the candle, but somewhat approximately equal in *all* directions. Accordingly at a fracture there is a good deal of "total reflection." The general attenuation downwards is doubtless partly due to defect of transparency, but also, and perhaps more, to the lateral escape of light at the surface of the candle, thereby rendered visible. By hindering this escape the brightly illuminated length may be much increased.

The experiment may be tried by enclosing the candle in a reflecting tubular envelope. I used a square tube composed of four rectangular pieces of mirror glass, 1 in. wide, and 4 or 5 in. long, held together by strips of pasted paper. The tube should be lowered over the candle until the whole of the flame projects, when it will be apparent that the illumination of the candle extends decidedly lower down than before.

In imagination we may get quit of the lateral loss by supposing the diameter of the candle to be increased without limit, the source of light being at the same time extended over the whole of the horizontal plane.

To come to a definite question, we may ask what is the proportion of light reflected when it is incident equally in all directions upon a surface of transition, such as is constituted by the candle fracture. The answer depends upon a suitable integration of Fresnel's expression for the reflection of light of the two polarisations, viz. :—

$$S^2=\frac{\sin^2(\theta-\theta')}{\sin^2(\theta+\theta')},\quad T^2=\frac{\tan^2(\theta-\theta')}{\tan^2(\theta+\theta')},\quad . \quad . \quad (1)$$

where θ, θ' are the angles of incidence and refraction. We may take first the case where $\theta>\theta'$, that is, when the transition is from the less to the more refractive medium.

The element of solid angle is $2\pi \sin\theta\, d\theta$, and the area of cross-section corresponding to unit area of the refracting surface is $\cos\theta$; so that we have to consider

$$2\int_0^{\frac{1}{2}\pi} \sin\theta\cos\theta\,(S^2 \text{ or } T^2)d\theta, \quad . \quad . \quad . \quad (2)$$

the multiplier being so chosen as to make the integral equal to unity when S^2 or T^2 have that value throughout. The integral could be evaluated analytically, at any rate in the case of S^2, but the result would scarcely repay the trouble. An estimate by quadratures in a particular case will suffice for our purposes, and to this we shall presently return.

In (2) θ varies from 0 to $\frac{1}{2}\pi$ and θ' is always real. If we supppose the passage to be in the other direction, viz. from the more to the less refractive medium, S^2 and T^2 being symmetrical in θ and θ', remain as before, and we have to integrate

$$2 \sin\theta' \cos\theta'\,(S^2 \text{ or } T^2)d\theta'.$$

The integral divides itself into two parts, the first form 0 to α, where α is the critical angle corresponding to $\theta=\frac{1}{2}\pi$. In this S^2, T^2 have the values given in (1). The second part of the range from $\theta'=\alpha$ to $\theta'=\frac{1}{2}\pi$ involves "total reflection," so that S^2 and T^2 must be taken equal to unity. Thus altogether we have

$$2\int_0^{\alpha}\sin\theta'\cos\theta'\,(S^2 \text{ or } T^2)\,d\theta'+2\int_{\alpha}^{\frac{1}{2}\pi}\sin\theta'\cos\theta'd\theta', \quad (3)$$

in which $\sin\alpha=1/\mu$, μ (greater than unity) being the refractive index. In (3)

$$2\sin\theta'\cos\theta'd\theta'=d\sin^2\theta'=\mu^{-2}d\sin^2\theta,$$

and thus—

$$(3)=\mu^{-2}\times(2)+1-\mu^{-2}$$

$$=\frac{1}{\mu^2}\left\{\mu^2-1+\int_0^{\frac{1}{2}\pi}\sin 2\theta\,(S^2 \text{ or } T^2)\,d\theta\right\}, \quad (4)$$

expressing the proportion of the uniformly diffused incident light reflected in this case.

Much the more important part is the light totally reflected. If $\mu=1{\cdot}5$, this amounts to 5/9, or 0·5556.

With the same value μ, I find by Weddle's rule—

$$\int_0^{\frac{1}{2}\pi}\sin 2\theta\,.\,S^2d\theta=0{\cdot}1460,\quad \int_0^{\frac{1}{2}\pi}\sin 2\theta\,.\,T^2d\theta=0{\cdot}0339.$$

Thus for light vibrating perpendicularly to the plane of incidence—

$$(4)=0{\cdot}5556+0{\cdot}0649=0{\cdot}6205;$$

while for light vibrating in the plane of incidence—

$$(4)=0{\cdot}5556+0{\cdot}0151=0{\cdot}5707.$$

The increased reflection due to the diffusion of the light is thus abundantly explained, by far the greater part being due to the total reflection which ensues when the incidence in the denser medium is somewhat oblique.

RAYLEIGH.

The Pressure of Radiation.

THE theory of radiation at present accepted is based on Maxwell's result that the pressure of any component frequency is one-third of its energy density, which appears to result from an assumption analogous to Boyle's law, according to which the excess pressure due to vibration, in the case of a gas, would be one-third of the energy density of the vibration.

Lord Rayleigh (*Phil. Mag.*, 1905) has shown that this cannot be true in the case of a gas, since the vibrations are adiabatic, and Boyle's law does not hold. For a monatomic gas, where the reasoning based on the kinetic theory is fairly certain, he deduces that the excess pressure should be two-thirds of the energy density.

In a recent note on radiation and specific heat (*Phil. Mag.*, October, 1913) I gave an outline of a new theory, showing good agreement with experiment, from which I deduced the result that "the total pressure of full radiation should be one-third of the intrinsic energy density, but this could not be true for

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[1] To what is the diffusion due? Actual cavities seem improbable. Is it chemical heterogeneity, or merely varying orientation of chemically homogeneous material operative in virtue of double refraction?

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the partial pressure of each component taken separately." So many of my correspondents, including Lord Rayleigh, have questioned the grounds of this statement, which is the crux of the whole problem, that it may be of interest to explain my reasons more fully.

Arguing on the analogy of a gas, it is evident that the vibrations of radiation must be regarded as adiabatic, and cannot satisfy pv=constant, unless the value of the index γ (the ratio of the total energy $E+pv$ to the intrinsic energy E) is equal to unity, which is impossible. We conclude either that the analogy is false, or that, if the vibrations are adiabatic, the ratio may have different values for different frequencies. Since the index γ is equal to $(E+pv)/E$, it is obvious that, to be consistent, we must have $p=(\gamma-1)E/v$, which agrees with Lord Rayleigh's result for monatomic gases, and may be true generally for the pressure of adiabatic vibrations.

According to my theory, radiation consists of the vibration of equal elementary units (Faraday tubes associated with ionic pairs) each possessing the same angular momentum, but having intrinsic energy proportional to the frequency ν and independent of the temperature T. The pressure is assumed to be equally divided between the molecular units according to the gas law $pv=RT$, because this gives the simplest possible explanation of the exponential term $e^{-b\nu/T}$ in the radiation formula as a direct consequence of Carnot's principle, and because equipartition of pressure is the most universal condition of equilibrium in physics.

It follows that the ratio of the pressure to the energy density, denoted by $\gamma-1$, must be of the form $T/b\nu$, which is different for different frequencies at the same temperature, but gives the mean value 1/3 for full radiation. The possibility of having different values for this ratio is explained by the fact that the vibrations are adiabatic, and the correction thus introduced into the theory of radiation is in this respect analogous to that introduced by Laplace into the Newtonian theory of the propagation of sound.

The assumption here made admits of a fairly simple experimental test, such as the following. Divide the radiation from a source, such as an arc light, into two parts of different frequencies. Compare the total energies and pressures by suitable means. The ratio of the pressure to the energy should be the same for each part on Maxwell's theory. On my theory, the part of lower frequency should have the higher pressure in a determinate ratio. I hoped to be able to try this crucial test before publishing even an outline of my theory, but the rapid extension of the Imperial College in recent years has left me insufficient leisure for so exacting an experiment, though it might not present serious difficulty to an expert in the measurement of radiation pressure.

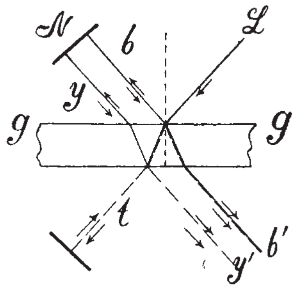
There are many other points in so brief a sketch which may require further elucidation, but these must be postponed. In the meantime I hope I have succeeded in demonstrating at least the possibility, if not the probability, of the fundamental assumption of my theory.

H. L. CALLENDAR.

Imperial College of Science, S.W., December 10.

Scattering in the Case of Regular Reflection from a Transparent Grating: an Analogy to the Reflection of X-Rays from Crystals.

1. *The Phenomenon.*—No doubt the following phenomenon has been noticed before, but I have seen no description of it. If a vertical sheet of white light L from a collimator is reflected from the two faces of a plateglass grating, having about 10,000 or more lines to the inch, g being the ruled face, the two beams b and y going to the opaque mirror N are respectively vividly blue and brownish-yellow. In other words, more blue light is regularly reflected from the ruled surface than is transmitted, and more reddish light transmitted than is reflected. Since the plate grating is not quite plane parallel, two of the four rays, b^1 and y^1, are seen in the same colours in the telescope. This is a great convenience in adjusting the displacement interferometer, where the spectra from b alone are wanted, and the y ray may be screened off at N, while the other y^1 has no spectrum.



The transmitted rays, t, after reflection show very little difference, the one reflected at g being perhaps slightly yellowish as compared with the other.

The spectra from b and y, if compared one above the other, are practically identical. The difference is not sufficiently marked to be discerned by the eye. Multiple reflection from the two faces gave no further results.

Finally, to be coloured blue, the beam must be reflected from the air side and not from the glass side, where but little appreciable effect is produced. If the grating is turned 180°, both the b and y rays are nearly white, while the t rays now correspond to the b and y rays, and are vividly coloured.

Outside the ruled surface and with any ordinary unruled plate of glass, all images are, of course, white. I mention this merely since one might suppose the absorption or colour of the glass to have something to do with the experiment. The film grating, where sharp reflection takes place from the glass and not appreciably from the film, does not show the phenomenon.

2. *Explanation.*—Scattering is usually and perhaps essentially associated with diffuse reflection. The present phenomenon, however, is strictly regular reflection—*i.e.* there is a wave front, for the blue and yellow slit images are absolutely sharp in the telescope. This is the interesting feature of the phenomenon, which associates it at once with the recent famous discovery of Friedrich, Knipping, and Laue relative to the reflection of X-rays from the molecules of crystals, and it is for this reason that I direct attention to it.

In case of the grating the sources of scattered light waves are not only identical as to phase, but these sources are at the same time equidistant. Hence collectively they must determine a wave front of somewhat inferior intensity, but otherwise identical with the wave front of normally reflected or diffracted light—*i.e.* the wave fronts of regularly reflected and scattered light are superposed.

Moreover, if the grating is turned in azimuth even as much as 45° on either side of the impinging beam (after which the many reflections and diffractions seriously overlap), the blue and brown colorations are distinctly intensified. This also is in accordance with anticipations; for the number of lines which are comprehended within the lateral extent s of the narrow beam L, as the angle of incidence i is varied, increases as $s \sec i$; whereas the lateral extent of the reflected beam is no larger than that of the impinging beam. Hence there should be increased intensity of scattered light in the ratio of $\sec i$, or increasing markedly with i from 1 for $i=0°$, to ∞ for $i=90°$. In other words, the scattering lines of the grating are virtually more densely disseminated when i increases.

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